ABSTRACT
A series of aero-optics tests have been carried out at Mach 7 in the Hypervelocity Wind Tunnel 9 (Tunnel 9) at the Arnold Engineering Development Center (AEDC). The test-bed used for the measurements were two flat plates which had sapphire windows mounted in titanium frames. Aero-optic measurements included near-field phase and intensity measurements made with two wavefront sensors, far-field point spread functions made with an imaging camera, and high frequency optical tilts (bore sight error) made with an X-Y Detector. Ancillary measurements of pressure and heat transfer on the test-bed plates were also made. The aero-optic measurements coupled with a variety of computations resulted in phase and intensity maps, bore sight errors, contained energy diameters (CED’s) and point spread functions (PSF’s). Comparisons between the various measurements are made to ascertain aerodynamic effects, instrument errors, facility-induced errors and measurement uncertainties.

INTRODUCTION
Hypersonic missile interceptors generally have a very wide flight envelope and, as a consequence, maneuver over a wide range of flight conditions. The combination of high velocities and low altitudes result in a severe heating of exterior surfaces. For those interceptors utilizing optical based seeker systems, the hypersonic flowfield over the optical window coupled with the aerodynamic heating of the windows can result in severe optical aberrations (distortion or blurring...
along with a tilt or bore sight error) of the images being detected by the on-board optical seeker. These aberrations are caused by the index of refraction fluctuations in the flowfield, window boundary layer and/or the distortion of the window. Density gradients across the bow shock and the compressed flowfield create refractive index variations and act as a gradient index lens. A combination of all of these effects lead to temporal and spatial variations in the index of refraction which in turn cause aberrations to the optical wavefront propagating through the flow. All of these effects and aberrations are often called aero-optics. Since it is the image of a target that is important in a seeker system, the measure of the image degradation is very important. Missile systems designed for hit-to-kill demand a very high precision in determining image centroids, image tilts (bore sight errors), and a variety of other optical image factors. The optical aberrations can add a large uncertainty to the high accuracy demanded by these missile systems. As a consequence, it is imperative that the flowfield effects such as density gradients throughout the flowfield, the state of the boundary layer, the optical window system distortion, and vibration effects are accurately modeled. In order to minimize risks in developing a hit-to-kill system, the thermal and structural response of the optical window and the optical aberrations induced by the hypersonic flowfield should be assessed through ground testing which duplicates true flight conditions as close as possible. To this end, a series of tests have been carried out in Tunnel 9 with a suite of optical instruments designed to determine the aero-optic effects on a seeker in a missile under true flight duplication.

FACILITY DESCRIPTION

Tunnel 9 is located at the White Oak, MD site of AEDC. It provides clean, uniform aerodynamic flowfields at high Reynolds numbers with long run times. Tunnel 9 has played a major role in the testing of reentry systems, endo-atmospheric interceptors, and aerospace plane programs. Specific testing areas include aerodynamics, high-speed inlets, aerothermal heating, jet interaction, and shroud removal.1,2

Tunnel 9 is a blowdown facility that currently operates at Mach numbers of 7, 8, 10, 14, and 16.5. Tunnel 9 uses pure nitrogen as the working fluid. The test section is over 12 feet long and five feet in diameter, which enables testing of full-scale model configurations. A layout of Tunnel 9 is shown in Fig. 1. Ranges for Reynolds numbers and supply conditions are listed in Table 1.

During a typical run, a vertical heater vessel is used to pressurize and heat a fixed volume of nitrogen to a predetermined pressure and temperature. The test section and vacuum sphere are evacuated to approximately one mmHg and are separated from the heater by a pair of metal diaphragms. When the nitrogen in the heater reaches the desired temperature and pressure, the diaphragms are ruptured. The gas flows from the top of the heater, expanding through the contoured nozzle into the test section at the desired test conditions. As the hot gas exits the top of the heater, cooler nitrogen from three pressurized driver vessels enters the heater base. The cold gas drives the hot gas in a piston-like fashion, thereby maintaining constant conditions in the test section during the run. A complete description of the current Tunnel 9 capabilities can be found in Reference 2.

This test series was performed in the Mach 7, thermal/structural test capability, which duplicates the harsh environments experienced by endo-atmospheric interceptors. Run times on the order of un-shrouded end game times at full flight duplication provide the thermal exposure needed for full-scale seeker window testing. The Mach 7 facility utilizes the high pressure and temperature capability in the Mach 14 heater but expands the flow to a Mach number of 7, concentrating the high enthalpy flow in a smaller, high energy nozzle core flow. This Mach 7 flow maintains high pressure and temperature providing full flight duplication. Table 2 lists the performance parameters for the Mach 7 facility. References 2 and 3 provide a complete description of the Mach 7 facility and calibration.

OPTICAL SUITE

A suite of instruments has been developed which provides a variety of optical measurements relevant to assessing the influence of hypersonic flows on aero-optic parameters. As was mentioned earlier, the image of a target on the focal plane array of a missile seeker is of primary interest. For a point source at infinity, the image measured at one focal length away from a lens is defined to be the point spread function (PSF). A perfect optical system with no aberrations, would have an Airy pattern for a PSF. Since the PSF is an indicator of how well the light can be focused through an optical system, it is important to understand and quantize as much information as possible from the PSF. This is a relatively easy measurement to make. However, the PSF measured in this manner is an absolute measurement. That is, all aberrations in the optical system, whether actual optical components or aero-optics effects, are manifested in the PSF and there are no viable means to separate them. From a Fourier optics view, the PSF is the real value of the complex field of the light incident on the final imaging lens, and as such, does not contain any phase information about the light field. The use of an instrument such as a Shack-Hartmann wavefront sensor provides a means to measure both irradiance and phase distributions of a light beam. To this end, Shack-Hartman sensors were used in our optical suite to provide this additional
information. A schematic of the suite is shown in Fig. 2 (CW setup) and Fig.3 (pulse setup). Details of the various setups will be described in the following section. The suite consists of two 2-D wavefront sensors (WFS), an imaging camera or point spread function camera (ICS) and a 2-D X-Y Detector and an assortment of mirrors, beam splitters, and dichroic mirrors to direct a 1-inch diameter beam through the facility and onto the sensors. The beam is reduced in size by a 5:1 telescope on the receiving table to reduce the beam size to match the detector size of the various sensors. The telescope is also set up to re-image the center of the wind tunnel on to the sensors. This assures that the sensors are detecting phenomena at a common reference plane. Descriptions of the individual sensors will now be given.

**Imaging Camera System**

The Imaging Camera System (ICS) consisted of an Adimec Model MX12P 1024x1024 pixel CCD camera along with a data acquisition system and computer with analysis and control software. The specifications for the camera are summarized in Table 3. The imaging camera measures directly the point spread function of the beam transmitted through the optical system. This is accomplished by focusing the beam through a long focal length lens (2000-mm) onto the camera’s focal plane array. The nominal point spread function (PSF) diameter is 82 pixels at the 84% Contained Energy Diameter or 615 um. The CCD has anti-bloom so the fill factor is roughly 30%. The centroid displacement, hence the bore sight error, of the image spot may be determined by appropriate calibration. Utilizing recently developed codes, such parameters as contained energy diameters, Strehl ratio, and blur may be determined from the PSF measured by the imaging camera and the computed PSF’s from the wavefront sensors. The camera operates at a nominal rate of 30Hz but can also be operated asynchronously at lower rates.

**Wavefront Sensors**

The two wavefront sensors used in this program are based on the Shack-Hartmann concept. They have several key components. These include a lenslet array and CCD camera as shown in Fig. 4. Light that is incident on the lenslet array is broken up into a number of individual beams, each of which focuses on the detector at a different location. Since light always propagates normal to its wavefront, the position of the focal spot is a measure of the average wavefront slope over the lenslet sub-aperture. By summing over all of the subapertures, the phase and intensity of the beam may be determined. This acquired data together with specifically developed software is used to compute a variety of parameters. This includes the following: the reconstructed wavefront from the slope measurements, the average tilt of the wavefront, rms wavefront error and wavefront curvature. In addition, Zernike polynomials are computed, and, the far-field PSF’s determined from which the centroids of the PSF’s are computed.

A definition of terms will be pointed out. For the wave-front sensors, the average wavefront slope of all the subapertures will be defined as the average tilt. When the far-field PSF’s are computed from the near-field 2-D phase and intensity distributions, the resulting displacement of the peak value from some initial value will be called the bore-sight error (BSE). The angular displacement of the PSF for the ICS will also be referred to as the bore-sight error.

The wavefront sensors used in these experiments are differential devices, that is they determine the phase of a test wavefront by calculating its departure from a predetermined reference wavefront. This feature gives the WFS much more flexibility in data analysis than the ICS, which, because it records only the intensity of the PSF resulting from the entire optical path, must be termed an “absolute” measuring device. The WFS also measures the phase changes resulting from the entire optical path if an “absolute” reference wave is used, i.e. a reference that is a collimated (plane) wave. However, local reference waves can also be used (i.e. the wavefront passing through the entire optical system just before tunnel flow begins – the tare data) that will result in the measurements of “relative” phase changes. In this case, if data taken during the tunnel run is compared to the tare data, the resulting WFS output will be phase changes caused only by the flow field. Phase contributions from the rest of the optical system will be effectively nulled out by the relative nature of the reference wave.

The two wavefront sensors used in these tests were manufactured by WaveFront Sciences and were models 6701 and 9701. The 6701 has a better spatial resolution and a 60Hz framing rate while the 9701 has less resolution and a 30Hz framing rate. Both can be operated asynchronously at lower frame rates for use with pulsed lasers. Pertinent details are shown in Table 4.

The Shack-Hartmann wavefront sensor measures both aspects of light: irradiance and phase distributions. Given this information in the near-field (i.e., incident upon the final imaging lens), the PSF can be computed. The far-field amplitude is given by (where u and v are the far-field angles):
\[ U(u,v) = \int_{-a/2}^{a/2} \int_{-b/2}^{b/2} U(x,y) \exp \left[ -\frac{2\pi i}{\lambda} (ux + vy) \right] \, dx \, dy \] (I)

Where \( U(x,y) \) is the near field complex field given by:

\[ F(x, y) = \sqrt{I(x, y)} \exp \left( \frac{2\pi i}{\lambda} \phi(x, y) \right) \]  

and \( I(x,y) \) is the measured irradiance distribution and \( \phi(x,y) \) the wavefront phase distribution. Note that Eq.1 represents the Fourier transform of the input field \( U(x,y) \) with respect to \( u \) and \( v \). Measuring the near field irradiance and wavefront distributions, and then computing the PSF, has a number of advantages over direct PSF measurement:

- The wavefront sensor’s relative mode of operation can be used to estimate effects induced by the flow, rather than that of the total optical system.
- The flow-induced aberrations can be directly determined by examining the wavefront.
- The image size and structure can be determined at an arbitrary plane in the optical system by Fresnel propagation analysis.
- Zernike and other aberration coefficients can be determined to allow characterization in terms of common optical system errors.

The two wavefront sensors used in these experiments have different size CCD arrays in the cameras. The 6701 has a significantly smaller array. As a consequence, the image overfills the 6701 array. This then precludes the use of the 6701 phase and intensity data to compute accurate PSF’s. Therefore, in general, only 9701 data will be shown in this paper.

**X-Y Detectors**

An X-Y Detector was included in the optical instrumentation suite on the receiver table during the experiments. On-Trak Photonics, Inc. made the X-Y Detector unit and consisted of a Model 2L4SP duolateral, two-dimensional, X-Y detector and a Model OT-300 position sensing amplifier. A duolateral detector sensor is a silicon photodiode that, using the lateral photoeffect\(^7\), detects the two-dimensional position of a light spot, the centroid of the power density, falling on the active area of the sensor. The Model OT-300 position sensing amplifier has transimpedence amplifier circuits and signal processing electronics that convert the photocurrents to voltages, and then amplify and process the voltages to produce normalized X position, Y position, and Sum voltages as outputs. The Sum output voltage is linearly proportional to the power in the incident light beam.

The sensor has an active area of 4.0 mm by 4.0 mm, a frequency response bandwidth of 4.4 MHz, and a position resolution of better than one part in one million.

The detectors measure the image centroid displacements with a high frequency response (up to 1.5kHz). Since the WFS and the ICS record data at about 30Hz, the data from the X-Y detector is used to correlate vibration effects on the aero-optic data. Due to the operational characteristics of the detectors, a CW laser source must be used.

**TEST MATRIX**

The test matrix consisted of two different scenarios. The first used continuous wave (CW) lasers for the optical sources; the second used a pulsed laser for the optical source. When using the CW source, the aero-optics data would have the character of being “time-averaged” since the acquisition time for the cameras for the ICS and the two WFS’s instruments was of the order of one millisecond. When using the pulse laser, the aero-optics data were in effect, instantaneous, since the laser pulse length was about 8 nanoseconds, there being one laser pulse per camera frame. In order to utilize effectively the various detectors, which had quite a different range in sensitivities, two different lasers with different wavelengths are used for each scenario. In the CW case, a red laser diode was used for the two wavefront sensors and the ICS, while a CW Nd:YAG laser was used for the X-Y position detectors. For the second scenario, a pulsed Nd:YAG laser was used for the wavefront sensors and the ICS while a CW He-Ne was required for the X-Y detectors. For both scenarios, framing rates for the wavefront sensors and the ICS were 25Hz. In all cases the individual data frames were linked to IRIG times so accurate correlations could be made for the data. For both cases the 2 beams were 1-inch diameter and co-aligned with each other.

Although several test entries have been made in the development of the sensors in this program, only three runs from the last entry will be discussed in this paper. The three runs are labeled 2690, 2692 and 2693. Table 5 summarizes the pertinent details. A pulse laser was used to obtain the data from the two WFS and the ICS in Run 2690; a cw laser in the other two. Generally, boundary layer trips have been used on the models. This was done to prevent boundary layer transition from taking place over the windows and, as such, the trips resulted in the boundary layers being totally turbulent over the windows. A reevaluation of the conditions led to our removing the trips so that a laminar boundary occurred over the windows. This is the case for Run 2693.

During a typical run, 220 frames of data are taken at a rate of 25 frames per second. This results in approximately 9 seconds of data being taken. The data...
systems are started approximately 2 seconds before the diaphragms rupture and flow begins. The two seconds of data taken before flow starts are used as tare data. It takes approximately 1 second for flow to establish after it begins; there is about 3 seconds of flow with constant conditions; and finally about 2 seconds of flow shutdown. During flow startup and shutdown, the Mach number is relatively constant, although the pressure and temperature are not. Immediately after the diaphragms rupture, the supply gas is relatively cold and when it is expanded through the nozzle, the gas is supercooled to a state where condensation of the nitrogen takes place. This results in a fog of nitrogen crystals in the test cell. This condition typically lasts for about a hundred milliseconds and then the temperature rises sufficiently so the fog clears and the flow becomes isentropic. During the period of time between one and four seconds (approximately) the flow properties are relatively constant and generally this is referred to as “good flow”. The flow which occurs about ½ seconds before “good flow” begins and about two seconds after “good flow” ends is isentropic; it just does not have constant properties such as temperature and pressure.

Test Model

The selection of model geometry was made to replicate the physical parameters related to BMDO endo-atmospheric interceptor geometries using optical seekers and having uncooled windows. The selected testbed configuration was a 15-degree wedge configuration with a sapphire window in a titanium frame as the seeker window. Fig. 5 shows the test bed geometry while Fig. 6 shows the important features of the window/frame combination. Fig. 7 shows the testbed mounted in the Mach 7 facility. Shown also in the photo is the opposing flat plate used to introduce the target beam to the test section. An identical window and frame combination was used in the target beam source plate due to the high thermal loads in Tunnel 9. The setup of the hardware in the facility could be mounted with both plates at 0 degrees incidence to the flow for calibration purposes, or with one plate acting as the interceptor test bed inclined at 15 degrees. More details of the model can be found in Ref. 1.

ALGORITHMS TO QUANTIFY PSF

Measured Quantities

As was mentioned in the previous section, the aero-optics measurements from the WFS include a vast assortment of parameters. In the analysis of the PSF’s that are generated from the WFS and measured by the ICS, three quantities are of particular interest. They are: (1) Strehl ratio, (2) Contained Energy Diameter (CED) and (3) Bore Sight Error. The Strehl ratio of a PSF is traditionally defined for imaging systems as the ratio between the peak intensity of that PSF ($I_{PSF}$) and the peak intensity of a diffraction limited PSF ($I_{DiffLim}$) and is

$$\text{Strehl}_{p/P} = \frac{I_{PSF}}{I_{DiffLim}}$$

(3)

The Strehl of the PSF during the wind-off tare is used to define $I_{DiffLim}$.

The CED (often referred to as Blur diameter) is defined as the diameter (reported here in units of microradians) at which a specified fraction of the total energy of the PSF is contained. Three CED’s are reported here, 50%, 75% and 84%. The 84% CED coincides with the diameter of the first dark Airy ring of a diffraction limited PSF. The total energy of the PSF is needed to put the contained energy in units of percent. The total energy is determined from the PSF measured during wind-off. Because the CCD camera used in the ICS has a limited dynamic range (approximately 3 decades), all of the energy of the PSF cannot be measured. For a nearly diffraction limited PSF, approximately 4% of the total energy in the tails of the PSF drops below the noise-floor of the CCD. This energy is not discernable from the noise and is subtracted off with the background.

The Bore Sight Error (BSE) is defined as the vector perpendicular to the wavefront tilt (reported here in units of micro-radians) introduced by aero-optic effects and window heating. The BSE due to the optical train of the test set-up is measured during the wind-off condition and subtracted from the reported BSE data.

Data Reduction

The methodology of data reduction is to take the PSF measured with the ICS and pass this data through a smoothing and filtering routine to remove background noise. Then taking this PSF and the PSF computed by the WFS from the measured phase and intensity, pass these PSF’s through routines which calculate the Bore Sight Error, Strehl ratios and Contained Energy Diameters.

The measurement of a PSF requires a CCD with a large dynamic range to measure simultaneously the peak intensity at or near the centroid, and the diffraction rings of the PSF. Fig. 8 shows a log plot of calculated intensity distribution through a cross-section of the PSF formed by the Aero-optic test bed. Note that only 96% of the total energy is inscribed within the 4th diffraction ring, where the intensity is down by over 3 decades.

In order to observe the diffraction rings, the noise (dominated by readout noise for our 25 Hz frame
rate) must be suppressed, which is of nearly the same order of magnitude as the intensity signal after the 2nd diffraction ring. In addition, to estimate the contained energy correctly, the CCD background (offset + dark counts) must be carefully subtracted.

**Smoothing & Background Subtraction**

An adaptive filter implemented in MATLAB™ was used to smooth the PSF’s measured with the ICS. The objective of the smoothing filter is to reject noise while maintaining the shape of the underlying PSF intensity distribution. To do this we fit a second order polynomial to a 13x13 pixels sub-window of the PSF image. The value at the center of the window is computed using the parameters of a least-squares fit. The choice of the 13x13 pixels for the window is a compromise. If the window is too small, the noise will not be effectively filtered. If the window is too large, the PSF will then be distorted. For the 13x13 window, given the feature sizes expected, a second order polynomial is an excellent fit to the underlying local intensity distributions. The filter adapts to the local intensity distribution, rejecting intensity fluctuations of higher order than 2, while preserving intensity distribution of 2nd order and lower very well. The background is determined by fitting a 2nd order polynomial to a dark image. The dark image is subtracted from each measured frame.

Fig. 9 shows a raw PSF image collected with the ADIMEC CCD. At this scale the background of the measurement obscures the details of the PSF. Fig. 10 shows the PSF with the background removed. The noise remains, obscuring the underlying diffraction rings of the PSF. Fig. 11 is the PSF after removing the background and smoothing with the polynomial filter described above. To ensure that the smoothing algorithm did not perturb the form of the PSF, the residual was observed to ensure that the difference between the original PSF and smoothed PSF was random. The size of sub-window used for smoothing (in this case 13x13) should be as large as possible to best reject noise, but not so large that it distorts the underlying distribution of intensity. The spot size was computed to be about 1.2 times the diffraction limit. Fig. 12 shows a section through a measured PSF in which the region justs outside the central lobe has been magnified. One sees that the smooth PSF overlays the noisy measured PSF without distorting the overall shape of the intensity profile.

**Strehl Calculation**

The Strehl can be computed in two different ways: (1) it can be computed from the ratio of the PSF peak intensities (which was described previously), and (2) it can be computed from the wavefront error. In the first method, the Strehl is computed from the ratio of the peak intensity of the PSF in each frame to the peak intensity of the PSF measured during the tare frames (Eq. 3). Typically 20 to 30 frames are averaged to obtain an average peak intensity from the tare frames. This data can then be thought of as “relative” since the Strehl is referenced to the tare and optical system effects have been essentially nondimensionalized. Each measurement of Strehl for both the ICS and WFS is referenced to the average peak intensity during the wind-off tare using Eq. 3. In the second method of determining the Strehl value, the large-aperture approximation is used. This approximation assumes that the spatial scales of the aberrations are both random and small compared to the dimensions of the aperture, and can be computed from the equation

\[
\text{Strehl}_{\text{WFE}} = e^{-\left(\frac{\text{WFE}^2\pi}{\lambda}\right)^2}
\]

where WFE is the RMS of the wavefront error as measured by the WFS in waves. The value of Strehl computed in this fashion can be thought of as being “absolute” since all effects due to the optical system are retained in the computations. Typical peak-to-peak variation for the wavefront in the present experiment is about \(\frac{1}{4}\) wave. This is then about \(\lambda/16\) for the RMS. Applying the previous formula gives a Strehl of about 0.86 for the WFS. Fig. 13 compares the Strehl measured with a diffraction limited optical system, and an optical system that has 0.25 P-V waves of wavefront error. The diffraction- limited system has a Strehl of 1 corresponding to 0 wavefront error introduced by the wind tunnel. The slope of the Strehl with wavefront error is near zero as the wavefront error approaches zero. The slope slowly increases (in the negative direction) and is nearly linear starting at wavefront error of approximately 0.25 P-V. A Strehl loss of approximately 0.19 is expected for a 0.25 wavefront error. If the optical system has a 0.25 wavefront error bias (before the wind tunnel is turned on), then the slope of Strehl with wavefront error is much larger. Here we have normalized the curve to equal 1 at the zero wavefront error of 0.25, as is typically done. Under these conditions, less wavefront error is required to decrease the Strehl by a significant amount. For a wavefront error of 0.25 waves introduced by the tunnel, the Strehl decreases by 0.46. The Strehl loss is over-reported by a factor of nearly 2.5. The reason for this error is the nonlinear relationship between wavefront error and Strehl ratio. The initial wavefront error of the optical system used to measure Strehl has a significant impact on the error of the measurement. This error tends to cause the Strehl to be over-reported.
**Contained Energy Calculation**

Contained Energy is computed by performing integrals over circles of increasing diameter surrounding the centroid of the PSF. Fig. 14 shows the PSF’s for a tare frame and a wind-on frame for the ICS. The measured tare PSF was nearly a diffraction-limited beam (< 0.25 P-V wave-front error). Also shown in the figure are the integrated CED’s for the two PSF’s. Note that the derivative of the Contained Energy approaches 0 near 40 pixels. This position corresponds to the first dark ring of the Airy pattern. Doing a weighted sum over a rectangular area performs each integral. For example, the weighting factors for a 3.5 pixel radius is shown in Fig. 15. At the center of the circle the weighting factors equal 1. At the edges of the circle the weighting for each pixel corresponds to the fraction of the pixel that is inscribed by the circle. Thus, the CED’s are computed over circles of increasing diameter surrounding the centroid of the PSF. The maximum radius for the present data is 200 pixels for the ICS. The WFS has a different pixel scaling; the maximum diameter was 50 pixels.

**Bore Sight Calculation**

The Bore Sight Error (BSE) is computed as a difference in centroid location. Pixel centroids from a series of tare frames are averaged. This average is subtracted from the centroid of each data frame. The BSE result is converted to micro-radians by multiplying the pixel location by the wedge calibration factor. Each WFS centroid is defined as the location of the maximum value of the PSF. Convolving the central 101x101-pixel portion of the PSF with a 19 pixel-diameter circle function further refines the ICS centroid.

### WAVELENGTH SCALING

One of the key advantages of measuring the transmitted wavefront is that the results may be scaled to different wavelengths. In previous sections, we have shown that the point spread function may be calculated accurately from the irradiance and phase distributions. The phase was derived from the wavefront distribution measured with the WFS through the simple relation:

\[
\phi(x, y) = \frac{2\pi w(x, y)}{\lambda} \tag{5}
\]

Note that the wavefront \(w(x,y)\) is a measure of the optical path difference of the transmitted light. As described previously, this light is derived from a transmission through the flow, windows and other optical elements. The flow on/flow off effects may be directly evaluated by using the measurements immediately prior to flow to compute a reference that is subtracted from successive measurements. Thus the wavefront sensor provides the irradiance and wavefront distributions of the flow induced aberrations. If the optical path of these aberrations is relatively short\(^1\), then the wavefront distribution uniquely represents the optical errors induced by turbulence, window heating, shock waves or other flow induced effects. Scaling the point spread function to a new wavelength from the measured distribution at a different wavelength simply involves changing the wavelength in Eq. 5 to the desired calculation wavelength, and then computing the point spread function through the Fresnel propagation integral of Eq. 1.

This calculation is extremely powerful because it does not require the wavefront measurement to be made at the same wavelength as the desired imaging application. This allows wavefront measurements to give a scaled merit of performance for different wavelength bands. This can now be used for design and analysis of seeker optics at different wavelengths.

For example, Fig. 16 is a frame from a WFS measurement that includes a significant amount of turbulence. The RMS wavefront error for this frame is 0.35 µm. This is typical of the wind tunnel turbulence measurements during established flow. The wavefront was measured at 633 nm using the wavefront sensor. Since the total path through the wind tunnel flow was short (< 0.4 m), there is little opportunity for chromatic effects to affect the wavefront distribution. Hence the wavefront distribution is an accurate measure of the optical path difference through the flow, irrespective of wavelength.

The effect of this optical path error distribution on the resulting point-spread-function is dramatic; an example is shown in Fig.17. At short wavelength (633 nm), the 0.35 µm RMS OPD is a significant fraction of the wavelength. Hence the Strehl ratio is almost zero for this case. The resulting point spread function is reduced to a speckle pattern, with a very large blur diameter. For a longer wavelength (1.5 µm), the 0.35 µm RMS wavefront error is a much smaller fraction of

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\(^1\) The assumption of short optical paths is necessary because of coupling between the irradiance and the phase distributions. Flow field turbulence causes rapidly changing densities, which in turn, cause wavelength-dependent changes in the index of refraction. If this chromatic dispersion propagates through a long optical path length it will have a significant affect on the phase and irradiance in the observation plane. However, for short optical path lengths, such as this aero-optics application, this dispersion does not propagate far enough to affect the measured distribution. See References 8 and 9 for a thorough discussion of this argument.
the wavelength. Hence the Strehl ratio is much higher, and the pattern is a combination of reduced speckle and the Airy pattern distribution for 1.5 \( \mu \text{m} \) light. By 3.5 \( \mu \text{m} \) the RMS wavefront error is less than 1/10 wave, and thus the speckle is greatly reduced. The point spread function is dominated by the Airy pattern, with little effects of the turbulence evident.

This example shows one of the advantages of measuring the wavefront rather than the point spread function directly. All of these calculations are derived from a single measurement at 633 nm. The wavefronts for the optical system add linearly as various components are added. Thus the measured wavefront could be used as the input to a seeker optical system lens (or other elements) and the point spread function for the entire system determined at the various wavelengths of interest. To make these same measurements with only an imaging camera configured to capture the point-spread-function directly would require separate measurements at each wavelength. Furthermore, since the point spread functions cannot be added linearly, the point spread function would need to be measured again for each additional optical element. This greatly complicates the job of the system designer.

**RESULTS**

**Calibration of Optical Sensors**

A considerable amount of time was devoted to verifying that the WFS and ICS were giving compatible results. Lab experiments were carried out where the input beam had curvature and tilts imposed on the wavefronts and these were then verified. Only after a thorough calibration were the instruments taken to the wind tunnel. In order to calibrate all the above sensors in the wind tunnel, a specially constructed device was used. Its principal component consisted of a precision optical wedge that produced a known tilt. This wedge is placed in the beam path and is rotated with an electrical motor at a relatively low speed of approximately \( \frac{1}{2} \) rev/sec. The outputs from all the sensors are then monitored, recorded and analyzed. The output tilt from the wavefront sensors is verified against the known wedge tilt. For the ICS camera and X-Y detectors, the measured tilts are used to establish calibration factors. The wedge is placed in two different locations. The first is inside the test section near the model window; the second is placed after the reimaging telescope. Rationing the two calibration factors (before and after) provides the demagnification of the telescope. Three different optical wedges can be used providing beam tilts of 125, 250 or 500 micro-radians. Shown in Fig. 18 is a typical calibration of the ICS system utilizing the 125 \( \mu \text{radians} \) wedge. A least squares fit of the data gives a RMS error of 1.13 \( \mu \text{radians} \) and a calibration constant for the ICS system of 0.76265 \( \mu \text{radians/pixel} \). The two WFS instruments and the X-Y detectors obtained similar data. The nominal RMS tilt for all three systems was of the order of 1 \( \mu \text{radian} \).

**Run Data**

Since the WFS measures phase and intensity in the near field and by using the Fresnel propagation analysis to determine the size and features of the image in the far field, it is of interest to compare the tilts measured in the near field to the BSE measured in the far field. (Remembering that the average of the wavefront slope for the WFS is called tilt while the displacement of the PSF from some initial value is the bore-sight error (BSE)). Typical results are shown in Fig. 19 for R2690. It should also be noted that the data has been low-pass filtered (<2Hz) using a second order non-causal filter which has a 12db/octave roll-off. There are gaps in the data from the start of flow (time = 0 sec) to about 0.4 seconds. This is the period of initial start up of the tunnel when condensation is a factor and its effect on the data is an issue. It can be seen there is good agreement (15% difference) between these two sets of data. The agreement between the WFS tilt and the ICS BSE is excellent. Thus one can have a high degree of confidence the two instruments give the same results and that the near field measurements can be transformed to far field parameters.

The tilts/BSE measured by all three instruments for the three runs is shown in Fig. 20. First of all, the agreement is very good. One sees that after the flow starts, the BSE (x component) increases to approximately 40 \( \mu \text{radians} \) at which point it begins to decrease due to the drop in total temperature. The y component exhibits a little different behavior in that it decreases initially but after flow establishes the BSE begins to increase. The second important detail is the difference in the x tilts/BSE for Run 2693. The y tilts/BSE are all very similar in shape and magnitude. On the other hand, the x tilts/BSE exhibit a different character for Run 2693. It was concluded that because the boundary layer on the window is laminar, the heat transfer is significantly less for this run. Since the tilts/BSE are virtually unchanged from the tare data, one can conclude that there is little effect from the wind tunnel’s freestream, the boundary layer on the windows, and window bending due to heating. For Runs 2690 and 2692 there is some tilts/BSE as measured by the optical systems (approximately 40 \( \mu \text{radians} \) in x and 20 \( \mu \text{radians} \) in y). In general, the data from the X-Y detector is only used to provide data that is used to correlate vibration effects on the aero-optic data.

The data for the Strehl ratio is shown in Fig. 21 for the 3 runs for both ICS and WFS. It should be noted that the data in R2690 is corrected for the variation in laser power from the Nd:YAG laser. Since the laser power varies from pulse-to-pulse, a reference power meter is used to compensate for this variation. This
variation in power does not exist during Runs 2692 and 2693 since a CW laser is used during these runs. A reference power meter was used, but it indicated that the power was constant. The Strehl exhibits similar behavior for both ICS and WFS. Immediately after flow starts, the Strehl drops to almost zero. This is because the tunnel gas is very cold and condensation of nitrogen is taking place in the flow. As a consequence, the condensation scatters the light and thus severely attenuates the light entering the cameras. This condensation only lasts about a hundred milliseconds. Immediately upon the condensation clearing, the cameras begin detecting the laser light. The value for Strehl increases from about 0.5 to 0.7 during the time where the flow has constant conditions. After this period the Strehl begins to decrease. This decrease is presumed to be caused by the large increase in density in the tunnel flow and will be discussed shortly. We see that the Strehl for the WFS is generally greater than the Strehl for the ICS for runs 2692 and 2693. It is surmised that this is caused by the differences in the way the sensors measure the aero-optics parameters. During a frame of data from the CW runs, the camera for the WFS is on for about a millisecond. During that time, the camera is integrating the light input on the sensor, that is, it is integrating the phase and intensity. During this time period, the flow is sweeping through the laser beam and any disturbances, such as turbulence, passing through the beam cause aberrations to the beam. The phase and intensity fluctuations are being averaged by the WFS. This has the effect of averaging or smoothing out the phase and intensity. When the WFS PSF is calculated from this phase and intensity, the PSF appears more like a diffraction limited beam and as a consequence has a larger peak value. However, the ICS camera is averaging the PSF’s (which have jitter) that are being imaged on its CCD array and are being smoothed. The final result is that the peak of the ICS PSF is smaller than the peak of the WFS PSF. In addition, for the WFS, the turbulence is being averaged out, but large tilts such as those caused by window deformation will not be averaged. The influence of the window should then be measured while the influence of turbulence should be mitigated during a CW run. The Strehl data for R2690 (pulse data), on the other hand, is about the same for the both systems. In this case the laser pulse is very short (approximately 8 nsec) so the aberrations are effectively frozen in each frame of the two systems with the final result being the PSF’s are the same and thus the Strehl values are the same.

Shown in Fig. 22 are the results for the Strehl value when it is computed using the WFE in Eq. 4. One sees in the tares for the three runs that the value for the Strehl varies from 0.85 to 0.93. The wavefront typically is λ/4 or less. One can see that the temporal variation between the WFS Strehl measured from the wavefront error agrees very well with the Strehl measured from the PSF’s peak-to-peak ratio as measured by both the WFS and ICS. There is a slight offset but that is because of the “absolute” and “relative” nature of the measurements. One can see the versatility of the WFS to determine the aero-optics phenomena.

As a general observation, the laser beam in some cases is transmitted through and/or is reflected by over 20 optical elements. It is quite obvious that the quality of optics is very important and the setup must be done with great care to assure that the beam is as close to diffraction limited as possible. Typically, the spot size of the focused beam was about 1.2 times diffraction limited or better.

As mentioned earlier we see how the Strehl changed before and after good flow and it was surmised that the change in tunnel density was the major influence. Shown in Fig. 23 is a plot of Strehl as a function of tunnel density. Although there is scatter in the data, one sees a trend that the Strehl decreases with increase in density. This proposition is more convincing if one accepts the premise that the Strehl has a value of 1 at zero density. This remains to be verified with additional data.

The results from the contained energy diameter calculations (CED) for the 50%, 75% and 84% contained energy diameters are shown in Fig. 24 for both ICS and WFS. Again we see excellent agreement. We see that during the run, the CED is fairly constant with substantial increases in the CED before and after good flow. This is due to the large increase in density during this time that result in the spot size getting larger and the value of Strehl going down. This is borne out in Fig. 25 that shows the relation between Strehl and the contained energy diameter for the 75% contained energy diameter. We see that the Strehl decreases monotonically as the contained energy diameter increases. This is consistent since one expects that if the peak of the PSF goes down, then the energy is spread out more, thus increasing the contained energy diameter.

Finally, shown in Fig. 26 are PSF’s from both the ICS and the WFS for identical frames. The times are –1.3 sec (tare frame #4), 1.8 sec (frame #80 - the beginning of “good flow”) and 4.0 sec (frame #135 - near the end of “good flow”). Also shown are the corresponding values of Contained Energy Diameters and Bore Sight Error. There is very good agreement between the results. Again, we see results that indicate that the WFS and the ICS give very compatible and encouraging results, thus indicating we can easily relate the near field measurements of phase and intensity made with the WFS to the far field measurements of the PSF’s made with the ICS.

This good agreement between the WFS and the ICS shows that the wavefront sensor is a very useful
instrument that is capable of providing a large amount of quantitative information about the quality of the aero-optic beams being sensed by optical seekers. Previously, this information had to be provided by several instruments with a limited data rate such as imaging cameras and interferometers. Phase information could only be provided by interferometry and generally only a few interferograms could be obtained during a run. The wavefront sensor can obtain several hundred frames of data during a run and provide the results in real time. Work is currently underway to develop a high resolution WFS capable of operating at 60Hz framing rates which would more than double the quantity of data.

CONCLUSIONS

A series of aero-optics measurements have been carried out at Mach 7 at flight duplication conditions. The test-bed used for the measurements consisted of two flat plates that had sapphire windows. Aero-optics measurements included near-field phase and intensity measured with a recently developed wavefront sensor, bore sight error measurements with an imaging camera which measured the point spread function directly, and an X-Y detector. The aero-optic measurements coupled with a variety of computations resulted in phase and intensity maps, bore sight errors, contained energy diameters and point spread functions. PSF’s generated from the phase and intensity maps measured with the wavefront sensors compare very favorably with the PSF’s measured with the imaging camera. Tilts and bore sight errors from the two sensors show excellent agreement as do the Strehl and contained energy diameters. The results show that the WFS and the ICS give very compatible and encouraging results thus indicating we can easily relate the near field measurements of phase and intensity made with the WFS to the far field measurements of the PSF’s made with the ICS. In addition, we see that the wavefront sensor is a very useful and versatile instrument that is capable of providing a large amount of quantitative information about the quality of the aero-optic beams being sensed by optical seekers. Finally, from a small data set, it appears that a laminar boundary layer on the sensor window reduces the heating to the window resulting in smaller amounts of bore sight error.

REFERENCES

### Table 1. Tunnel 9 Nominal Capabilities

<table>
<thead>
<tr>
<th>Contoured Nozzle</th>
<th>P0 Range (atm)</th>
<th>T0 (K)</th>
<th>Re# Range (X10^6/m)</th>
<th>Run Time (seconds)</th>
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<tr>
<td>7</td>
<td>177-805</td>
<td>1932</td>
<td>12-52.5</td>
<td>1-6</td>
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<td>8</td>
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<td>0.33-5</td>
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<td>1006</td>
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<td>0.2-12.46</td>
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<td>1856</td>
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### Table 2. Mach 7 Thermal/Structural Facility Nominal Performance Parameters.

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<th>Supply Conditions</th>
<th>Test Cell Conditions</th>
<th>Performance</th>
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<tr>
<td>P0 (atm)</td>
<td>T0 (K)</td>
<td>Qinf (KPa)</td>
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<td>90-730</td>
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</table>

### Table 3. ADIMEC CCD Camera Specifications

- Resolution: 1024 x 1024
- Dynamic range: 12-bit
- Pixel Size: 7.5 µm
- Read-out rate (full image): 30 Hz

### Table 4. Wavefront Sensor Technical Specifications

<table>
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<tr>
<th>Parameter</th>
<th>9701 WFS</th>
<th>6701 WFS</th>
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<td>Framing Rate</td>
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<td>60Hz</td>
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<tr>
<td>Integration time</td>
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<td>1/32,000 s</td>
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<td>Lenslet array FL</td>
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<td>Areas of Interest</td>
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<td>49 x 40</td>
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### Table 5. RUN CONDITIONS

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<th>ANGLE OF ATTACK</th>
<th>BOUNDARY LAYER</th>
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<td>0</td>
<td>LAMINAR</td>
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Fig. 1 Tunnel 9 Layout
**PHASE 1b SOURCE TABLE SCHEMATIC - CW RUNS**

**PHASE 1b CW RECEIVER TABLE SCHEMATIC**

*NOTE: For CW these dichroics are Max T at 635nm for 45 deg. (Two required to prevent 532nm leakage.)*  
L1 f = 1500mm; L2 f = 300mm; L3 f = 2000mm; L4 f = 1000mm; L5 f = -250mm.

**Fig. 2 Schematic of CW Laser Optical Setup**
NOTE: 6'x2.5' Table with long axis parallel to Light Tube axis

LEGEND FOR YAG LASER OPTICS:
1. Dichroic: T@1.06; R@0.532
2. Beam Dumps (2)
3. Half-wave Rotator
4. Thin Film Polarizer
5. Focussing Lens (f=400mm)
6. Spatial Filter Pinhole
7. Collimating Lens (f=500mm)
8. Beam Sampler
9. Focussing Lens (f=50mm)
10. Apertures (2) (scrapers)

PHASE 1b SOURCE TABLE SCHEMATIC - PULSED RUNS

NOTE: For Pulse these dichroics are Max T @ 532nm at 45 deg. (Two required to prevent 532nm leakage.)
L1 f = 1500mm; L2 f = 300mm; L3 f = 200mm; L4 f = 1000mm; L5 f = -250mm.

PHASE 1b PULSE RECEIVER TABLE SCHEMATIC

Fig. 3 Schematic of Pulse Laser Optical Setup
Fig. 4 Wavefront Sensor Schematic
Fig. 5 Optical Testbed Installation.

Fig. 6 Window and Frame Design

Fig. 7 Model Installed in Tunnel

Fig. 8 PSF of diffraction limited spot for optical test set-up. Percentages show Contained Energy relative to PSF diffraction rings.

Fig. 9 Raw image of PSF

Fig. 10 Background removed from PSF, un-smoothed.

Fig. 11 Background removed, smoothed.
**Fig. 12** Section of the edges of a PSF (diffraction rings) of a measured raw PSF is shown (blue), along with the smoothed PSF (red).

**Fig. 13** Strehl measurement made when initial P-V wavefront error is 0 waves (red line), and when P-V wavefront error is 0.25 waves (blue line).

**Fig. 14** Contained Energy shown with the corresponding PSF, for a nearly diffraction limited PSF (measured during wind-off tare), and an aberrated PSF measured during wind-on. The PSF’s and the Contained Energies share the same scale since they both are normalized to 1.

**Fig. 15** Weighting factors for computing the Contained Energy for radius 3.5 pixels.
Fig. 16 – Wavefront error for pulsed wavefront sensor measurement. RMS wavefront error is 0.35 µm. The wavefront was measured at 633 nm.

Fig. 17 – Point spread function calculations at different wavelengths.

Fig. 18 Wedge Calibration of ICS

Fig. 19 X-TILT versus X-BSE for 9701 WFS
Fig. 20 Comparison of X and Y Bore-Sight Errors for the ICS, WFS and X-Y Detector for Runs 2690, 2692 and 2693
Fig. 21 Strehl Ratio Determined by Ratio of Peak Intensities of PSF’s

Fig. 22 Strehl Determined From Wavefront Error
Fig. 23 Strehl ratio versus Freestream Density
Fig. 24 Contained Energy Diameters for R2690
Fig. 25 Strehl versus Contained Energy Diameter for 75% Level
Fig. 26 Comparison Between Wavefront Sensor and Imaging Camera Systems for 3 Frames